Practicum 2: Computing

In this week's practicum we will be asking Python to do tedious computations for us. Specifically, we will compute basic ballistic trajectories for High Explosive rounds fired from a WWII 105mm Howitzer, under standard atmospheric conditions.

1 Ballistics Trajectories for a WWII 105mm Howitzer

A Howitzer is a classic piece of army artillery, used in WWII and other conflicts. These weapons fire high explosive shells at an upward angle, and have a range of several miles. To use such weapons in wartime, forces in the field needed to aim them. Artillery officers would make use of firing tables, which are summaries of the known ballistic trajectories for the particular weapon, shell, fuze, and charge. Experimental testing would be used to find the ballistic properties of a particular combination (ex. muzzle velocity, drag coefficient). From these, human "computers" would calculate out ballistic trajectories at different firing angles, under standard conditions, and then compute modifications for non-standard conditions. Generating a single firing table involved computing hundreds of ballistic trajectories, each of which would take a person with a desk calculator upwards of two work days.
The military need for many new firing tables for many new weapons in the lead-up to the US entry into WWII drove the invention of the ENIAC, the first modern computer. As such, automating the computation of ballistic trajectories was one of the very first tasks for the world's first computer programmers.

FIRING TABLES
FOR
HOWITZER, 105-MM, M2 AND M2A1
FIRING
SHELL, H. E., M1
WITH
FUZE, P. D., M48
FUZE, COMBINATION, 25-SEC.
TIME AND SUPERQUICK, M54

Prepared by the
Ordnance Department, U. S. A.
December 1941

UNITED STATES
GOVERNMENT PRINTING OFFICE
WASHINGTON : 1943

34
SHELL, H.E., M1
FUZE, M48, M54
M.V., 875 f/s CHARGE 4

PART 2d TABLE A

1	2	3	4	5	6	7	8	9	10	11	12	13
						Probable error					Deflection effect	
Range	Elevation	Fork	Change in elevation for 100 yd change in range	Change in range for 1 mil change in elevation	Time of flight	Range	Deflection	Height of burst	Slope of fall	Line number of metro message	Drift	Lateral wind of 1 MPH (+)
R	El	F	C	mil	Time	e_{pr}	e_{pd}		Slope	L	Dft	W–D
yd	mil	mil	mil	yd	sec	yd	yd	mil	1/–	No	mil	mil
0	0.4	2	6.2	16	0.0	7	0	0		1	0	0.0
100	6.6	2	6.2	16	0.3	7	0	0	154	1	0	0.0
200	12.8	2	6.2	16	0.7	8	0	0	77	1	0	0.0
300	19.2	2	6.4	16	1.0	8	0	0	51	1	0	0.0
400	25.6	2	6.4	16	1.4	9	0	0	38	1	0	0.0
500	32.0	2	6.4	16	1.7	9	0	0	30	1	0	0.0
600	38.6	3	6.6	15	2.1	10	0	0	25	1	0	0.0
700	45.2	3	6.6	15	2.4	10	0	0	21	1	0	0.0
800	52.0	3	6.6	15	2.8	11	0	0	18.5	1	0	0.0
900	58.8	3	6.8	15	3.2	11	0	1	16.4	1	0	0.0
1000	65.6	3	6.8	15	3.5	12	0	1	14.7	1	1	0.0
1100	72.4	3	6.8	15	3.9	12	0	1	13.3	1	1	0.0
1200	79.2	4	7.0	14	4.2	13	0	1	12.1	1	1	0.0
1300	86.2	4	7.0	14	4.6	13	0	1	11.1	1	1	0.1
1400	93.2	4	7.0	14	5.0	14	0	1	10.2	1	1	0.1
1500	100.4	4	7.2	14	5.4	14	0	1	9.5	1	1	0.1
1600	107.6	4	7.2	14	5.7	14	0	1	8.9	1	1	0.1
1700	114.8	4	7.2	14	6.1	15	0	1	8.3	1	1	0.1
1800	122.2	5	7.4	14	6.5	15	0	1	7.8	1	1	0.1
1900	129.6	5	7.4	14	6.9	16	0	1	7.3	1	1	0.1
2000	137.2	5	7.4	14	7.2	16	0	1	6.9	1	1	0.1
2100	144.8	5	7.6	13	7.6	17	0	1	6.5	1	1	0.1
2200	152.4	5	7.6	13	8.0	17	0	1	6.2	1	2	0.1
2300	160.0	5	7.6	13	8.4	18	1	1	5.9	1	2	0.1
2400	167.8	6	7.8	13	8.8	18	1	1	5.6	1	2	0.1
2500	175.6	6	7.8	13	9.2	19	1	1	5.3	1	2	0.1
2600	183.6	6	8.0	12	9.6	20	1	2	5.1	1	2	0.1
2700	191.6	6	8.0	12	10.0	20	1	2	4.8	1	2	0.1
2800	199.8	7	8.2	12	10.4	21	1	2	4.6	1	2	0.1
2900	208.0	7	8.2	12	10.8	21	1	2	4.4	1	2	0.1
3000	216.4	7	8.4	12	11.2	22	1	2	4.2	1	2	0.1

FT 105-H-3

1.1 Download

Please download `pr02.zip` from the practicum website (or Blackboard), unzip it, and move the directory/folder to where you want it in your file system. Use Atom's `File > "Add Project Folder..."` to open this folder and view `ballistics.py`.

1.1.1 Muzzle Velocity

Experimental tests show that the muzzle velocity of this weapons combination is `875 ft/s`. To use the ballistics equations, this needs to be converted in meters-per-second from feet-per-second.
You will need to multiply this value by `0.3048 m/ft` to convert it.

1.1.2 Forward-facing Area

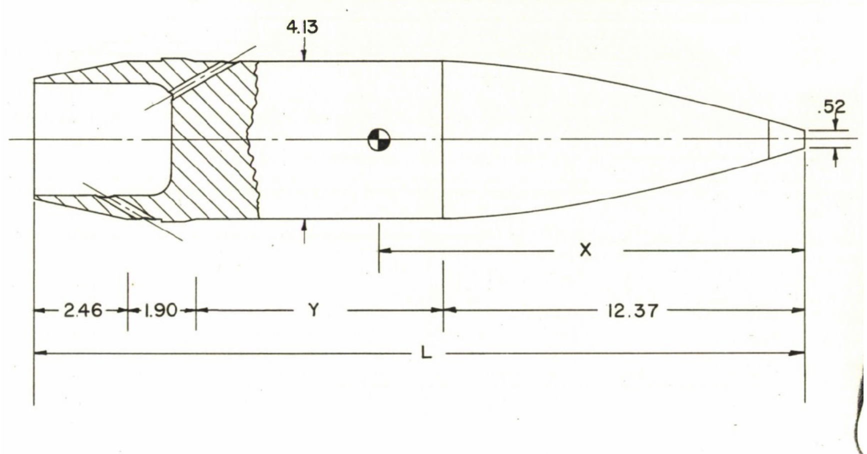
Besides the explosive charge and gravity, air resistance is the most important force acting on the shell as it moves through the air. First, we need to calculate the total area of a High Explosive shell that faces forwards as it moves through the air, given that the diameter of the round is `105mm`, or `0.105m`.
You will need this formula to calculate the forward-facing area:

$$A = \pi * \left(\frac{d}{2}\right)^2$$

Where:

- $\pi = 3.1415926536$

PHYSICAL PROPERTIES



TYPE	WT (LBS)	A (LBS-IN²)	B (LBS-IN²)	X CM(IN)	L (IN)	Y
1	29.07	72.60	919.0	13.78	23.22	6.49
2	28.76	71.40	892.0	13.59	23.00	6.27
3	29.20	72.60	885.7	13.45	22.84	6.11
4	29.08	72.27	861.0	13.29	22.63	5.90

NOTE: ALL DIMENSIONS ARE IN INCHES

FIGURE I

1.1.3 Terminal Velocity

Air resistance can be incorporated into ballistics trajectory equations by calculating the terminal velocity of the specific object, in this case a High Explosive shell.
You will need this formula to calculate the terminal velocity:

$$v_t = \sqrt{\left(\frac{2 * m * g}{\rho * A * C_d}\right)}$$

Where:

- A (in m^2) is the forward-facing area of the High Explosive shell
- $m = 13.19kg$ is the weight of the shell (from product specifications)
- $C_d = 0.0494$ is the coefficient modifting drag for the aerodynamic shape (from experiments)
- $\rho = 1.225kg/m^3$ is the density of air at sea level and 15 degrees C
- $g = 9.8m/s^2$ is the acceleration of gravity

Square root in Python There are two ways to have Python compute a square root.

- Use exponentiation to a fraction: `x**(1/2)`
- Import a function from Python's 'math' library using: `from math import sqrt`

1.1.4 Ballistic trajectory

Now that we have the terminal velocity of a High Explosive shell, we can compute the trajectory of one fired from a 105mm Howitzer. You'll want your program to define the initial conditions of the trajectory. Then have your program ask you for a time (in seconds), and return the position of the shell. In the table below, you can fill in what it gives you. Use your program to find where and after how long this shell would hit a target at different elevations. Try plotting the trajectory to see if it makes sense!

First define the initial conditions and constants:

- v_i, computed earlier
- v_t, computed earlier
- θ, the firing angle (which you are free to choose)
- $g = 9.8m/s^2$ is the acceleration of gravity
- $e = 2.718281828$ is the exponential constant

Then have the program ask you for a time input:

- t, in seconds, given by user

Now use these formulas to compute the shell's position at time t:

$$x_t = \frac{v_t}{g} * (v_i * cos(\theta)) * \left(1 - e^{\frac{-g*t}{v_t}}\right)$$

$$y_t = \frac{v_t}{g} * (v_i * sin(\theta) + v_t) * \left(1 - e^{\frac{-g*t}{v_t}}\right) - v_t * t$$

Finally have your program print the `x` and `y` position.

Cosine and Sine in Python You will want to import these functions from Python's 'math' library. To do so make sure to include `from math import cos, sin` somewhere in your code. These functions take angles in *radians*:

30°	0.5236
45°	0.7854
60°	1.0472

To fire your Howitzer at any angle, simply import the `radians` function from Python's 'math' library.

1.1.5 Is your code correct?

You can now use your program to trace out this ballistic trajectory!
As before, this command in the CLI will run your program: `python ballistics.py`
Try giving your code several values that make sense – a few seconds to a few tens of seconds. Use the table below to record the output. How long does it take for the High Explosive shell to reach the "ground"? You can use the blank space to plot your trajectory. Does it look like your code is correct?

t	$x(t)$	$y(t)$

1.2 Submit Your Work!

Please submit your work to Blackboard.
If you used one file, you can upload it as-is.
If you used more, please first make a `zip` file of the folder within which you did your work.

- On MacOS, select your work folder, right-click and choose `Compress pr02`
- On Windows, select your work folder, right-click, `Send To`, `Compressed (zipped) folder`

1.3 Just for fun

WWII 105mm Howitzers have have fallen out of fashion for military use, but they serve a few specialized purposes even today. For example, one can find them in use for avalanche control in and around ski areas.

Also, please don't use this exercise to make actual firing tables! We are still simplifying, especially in assuming a constant drag coefficient. Really, these shells are flying so fast that the speed of sound gets involved... the drag is not actually constant.

1.4 Citations

Kleiman, Kathy. 2016. Great Unsung Women of Computing. San Francisco, California, USA: Women Make Movies. https://link.ezproxy.neu.edu/login?url=https://northeastern.kanopy.com/node/2288448
Polachek, H. 1997. "Before the ENIAC [Weapons Firing Table Calculations]." IEEE Annals of the History of Computing 19(2): 25–30.
Boyer, Eugene D. 1958. DRAG AND STABILITY PROPERTIES OF THE 105MM SHELL, HE, T388: Fort Belvoir, VA: Defense Technical Information Center. http://www.dtic.mil/docs/citations/AD0305107
"Wolfram Demonstrations Project Brings Ideas to Life with over 11k Interactive #WolframNotebooks for Education, Research, Recreation & More. #WolframDemo #WolfLang."
http://demonstrations.wolfram.com/ProjectileWithAirDrag/
"Road Warriors- UDOT's Avalanche Battle on Utah's Canyon Highways."
http://utahadvjournal.com/index.php/road-warriors-udots-avalanche-battle-on-utahs-canyon-highways.

This handout was originally created by Carolina Mattsson, Fall 2019.